A STATISTICAL METHOD FOR ESTIMATING THE MEAN RELATIVE HUMIDITY FROM THE MEAN AIR TEMPERATURE

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ABSTRACT

This paper outlines a method of estimating the mean relative humidity from the mean temperature. Ordinary linear regression techniques are used, with a correction added to account for the systematic geographical distribution of the regression errors.

1. INTRODUCTION

The Institute of Atmospheric Physics of The University of Arizona, with the cooperation of the U.S. Weather Bureau, has recently published a comprehensive climatic summary for the State of Arizona [1]. This summary contains, among other information, estimated values of the mean monthly relative humidity at 0600 and 1800 MST for 113 cooperative weather stations in the State. It is the purpose of this paper to describe the method of estimation, which involves only the most elementary physical reasoning and statistical techniques. The simplicity of the method makes it equally applicable to arid and humid regions. However, no true reliability test can be presented, since all available data were used in determining the final relationships.

2. THE METHOD OF ESTIMATION

By making use of a simplified form of the Magnus equation presented by Holmboe, Forsythe, and Gustin [2], the author [3] has shown that an approximately linear relationship should exist between the common logarithm of the relative humidity and the air temperature. That is,

$$\log \hat{R}=c-dt, \tag{1}$$

where $\hat{R}$ is the relative humidity in percent estimated from the air temperature, t, in degrees Fahrenheit. The constants c and d are functions of the ratio of the dew point temperature to the air temperature, both in degrees absolute. Although this ratio varies only slightly, averaging about 0.95 in Arizona, even a change of 0.04 may double or halve the constant d. For this reason, and because of the absence of extensive dew point data, especially for cooperative weather stations, it was believed expedient to use least squares methods to determine the constants in equation (1). This approach has the advantages of (a) minimizing the sum of squares of the differences between observed and estimated relative humidities, and (b) yielding a measure of the goodness of fit, i.e., the correlation coefficient. It also does not directly involve any of the assumptions made in setting up equation (1).

In this study, common logarithms of the average monthly 0600 and 1800 MST relative humidities at all Arizona stations for which they are available were correlated, respectively, with the average monthly minimum and maximum temperatures. The failure of the times of these extremes to coincide exactly with 0600 and 1800 MST has no great bearing on the problem, although the resulting regression coefficients may be quite different from those expected from purely mathematical reasoning. These coefficients and the correlation coefficients for each month are listed in table 1. The sample size used varied between 21 and 22; i.e., there were at least 21 stations in the State in each month for

TABLE 1.—*The regression coefficients c and d in the expression $\log \hat{R}=c-dt$, relating the estimated average relative humidity, $\hat{R}$, in percent, to the average temperature, t, in degrees Fahrenheit. At 0600 MST, the temperature is the average minimum; at 1800 MST, it is the average maximum. Also given is the correlation coefficient, r, between the common logarithm of the relative humidity and the temperature. All coefficients were determined from data for 23 Arizona stations*

Month	Hour					
	0600 MST			1800 MST		
	c	d	r	c	d	r
January	1.936	0.00415	−0.71	2.142	0.00894	−0.87
February	1.956	0.00425	−0.78	2.174	0.00963	−0.87
March	1.960	0.00519	−0.80	2.232	0.01135	−0.82
April	1 974	0.00582	−0.77	2.219	0.01122	−0.82
May	1.988	0.00659	−0.74	2.168	0.01076	−0.74
June	1.935	0.00533	−0.61	2.032	0.00826	−0.55
July	2.097	0.00500	−0.83	2.596	0.01142	−0.84
August	2.090	0.00394	−0.80	2.434	0.00931	−0.82
September	1.995	0.00350	−0.70	2.157	0.00723	−0.70
October	1.988	0.00458	−0.78	1.986	0.00562	−0.64
November	1.943	0.00488	−0.75	2.016	0.00643	−0.72
December	1.941	0.00402	−0.73	2.049	0.00664	−0.75

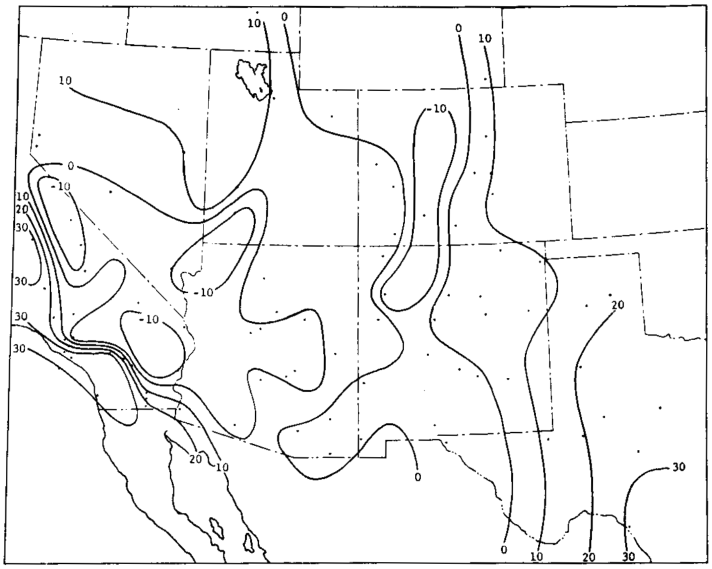

FIGURE 1.—Deviations of the observed average 0600 MST relative humidities for the southwestern United States in April from the values estimated by equation (1). The regression constants were determined from data for 22 Arizona climatological stations.

which both mean relative humidity and mean temperature data were available. With these sample sizes all correlations in the table differ significantly from zero at the 1 percent level of confidence.

In general, the best results, i.e., the highest correlations, were obtained for the winter and summer months of high humidity and the poorest results for the spring and fall months of low humidity. The standard error of estimate, not shown in the table, averages about 6 percentage units of relative humidity, ranging from about 4 to 8 percentage units. It has no systematic variations, since the months with the best correlations between temperature and relative humidity are also the months of greatest variance of these variables.

In using equation (1) to estimate the mean monthly relative humidity at stations for which only temperature data are available, it is convenient to plot it on semi-log paper, with relative humidity on the logarithmic scale. When this is done for each set of constants in table 1 a series of 24 straight lines results, one for each of the 2 hours in each of the 12 months. It is then merely necessary to enter these graphs with the average maximum or minimum

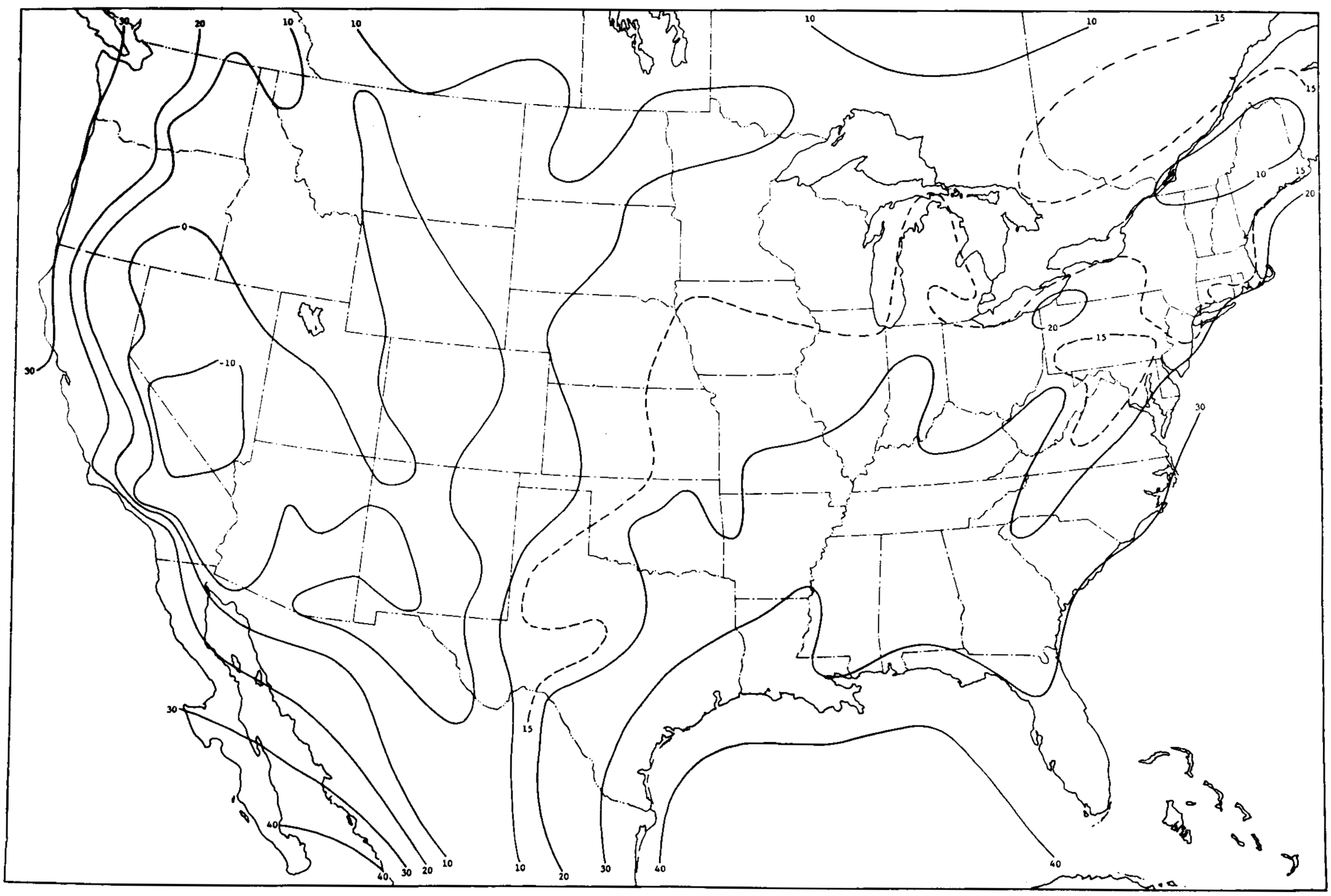

FIGURE 2.—Deviations of the average annual relative humidities (the means of the highest and lowest reported hourly values) for the contiguous United States, southern Canada, and northern Mexico from the values estimated by equation (2), using the average annual temperatures as the predictors. The regression constants were determined from data for 21 Arizona climatological stations.

temperature and read off the estimated average 0600 or 1800 MST relative humidity for a particular month.

While these estimates are probably fairly accurate, they may be improved upon by noting that the errors of estimation at the stations from whose data the regression constants were evaluated have a definite geographical pattern. This pattern may be analyzed to give errors of estimate for any station in the area for either hour and for any month. As an example, figure 1 shows the deviations of the observed relative humidities for April at 0600 MST from the values estimated using equation (1). In practice only the State of Arizona was considered. However, here the error analysis has been extended to all of the Southwest, using average relative humidities obtained by the author [3] in order to bring out more clearly the geographical distribution of errors. When these deviations from regression, denoted by e, are taken into account, equation (1) becomes

$$\hat{R}=\exp\ [2.3(c-dt)]+e$$

which is the expression used to estimate the monthly 0600 and 1800 MST relative humidity at 91 cooperative weather observing stations in Arizona. The first term on the right was evaluated from the graphical representation of equation (1) using the constants of table 1; the second term was determined for each station, hour, and month from analyzed state maps of the deviations from regression, i.e., deviations from equation (1).

Figure 1 has a definite climatological interpretation insofar as it delineates regions of moisture deficit and surplus in the Southwest. Thus, a station on the southern California coast recording the same average 0600 temperature in April as a town in central Arizona might be expected to have an average relative humidity more than 20 percentage units higher than the Arizona town. The same would be true for a city in southern Texas, another region of moisture surplus (relative to central Arizona). On the other hand, the Mohave Desert, southern Nevada, and the central Rocky Mountains have a moisture deficit.

For the year as a whole, the regression equation relating the logarithm of the average annual relative humidity (the mean of the 0600 and 1800 MST values) to the average annual temperature has the following form:

$$\log \hat{\bar{R}}=2.042-0.00621\bar{t} \tag{2}$$

The correlation coefficient between the two quantities is 0.90; the standard error of estimate of the relative humidity is about ± 3.4 percent. This equation, derived from Arizona data, was applied to all first-order Weather Bureau stations in the contiguous United States, Alaska, Canada, and Mexico, using the average of the highest and lowest reported hourly mean annual relative humidities for $\bar{R}$.

The distribution of the errors of regression for the contiguous United States, southern Canada, and northern Mexico is shown in figure 2. Largest positive values, exceeding 40 percentage units, are found in the Caribbean Sea and the Atlantic and Pacific Oceans. Negative errors of regression are common only in the southern Great Basin, the Rocky Mountain system, and the Chihuahuan Desert of Mexico. In the eastern United States, where the pattern appears to be disturbed only by the Great Lakes, the Mississippi River, and the Appalachian Mountains, values range from about 12 percentage units along the northern border to over 35 units in southern Florida.

From figure 2 and equation (2) it is possible to estimate the mean annual relative humidity at any point in the country given its mean annual temperature. This estimate should be better than that derived from a map of mean annual humidity, because the latter varies greatly both horizontally and vertically, while the regression deviations are relatively insensitive to changes in topography, these changes being taken into account mainly by variations in the mean annual temperature.

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